IDEAL HT: A HOUGH TRANSFORM ASSESSMENT TOOL

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Abstract: This paper addresses a method for the calculation of the ideal Hough Transform (HT) data of a segment. The calculation only depends on the basic physical facts (i.e. the endpoints of the segment), and therefore is free from the effect of discretisation errors in both image space and parameter space. The rounding errors in the voting process are also eliminated. The ideal HT data obtained is employed to evaluate the HT data generated by other HT varieties and alleviates the difficulties in assessing those HT algorithms. The performance of standard HT (SHT) is analyzed as an example.

Key words: Hough Transform, Hough Butterfly, Segment Detection.

1. INTRODUCTION

HT [1] is a useful tool for object detection. Straight segment detection is one of the most successful applications of HT. The straight line detection using HT is based on mapping the image space feature points (x,y) to the normal parameter space (i.e. (θ,ρ) space) by the following equation

$$\rho = x\cos\theta + y\sin\theta. \tag{1}$$

However, the HT does not directly detect segments, but just maps them to the parameter space and generates HT data. The peaks in HT data present the normal parameters of the straight lines containing the segments. The HT is robust when used for straight line detection, i.e. only detect θ and ρ, because it converts the straight line detecting problem to a peak seeking problem. However, when it is employed to detect the complete parameters of segments, such as (θ,ρ), width, length, end points, even collinearity and continuity, the robustness is seriously affected. To detect the complete segment parameters, a variety of HT based methods were presented [2–7], among them the butterfly based methods got more and more attention [3, 4, 6]. The butterfly based segment detection is based on the fact that Hough transform is tightly related to the Radon transform [8]. Therefore, detailed information about image structure, including detailed information about line segments, exists in the Hough space. However, these details are presented not only by the peak but also by other HT cells around the peak, i.e. the butterfly. Thus the quality of HT butterfly directly affects the performance of segment detection. Unfortunately, the approach for assessing the HT butterfly quality is still an open problem. Furthermore, besides the standard Hough Transform (SHT), many HT varieties were proposed such as probabilistic Hough(PHT) [9], Randomised Hough Transform (RHT) [10], Fast Hough Transform(FHT) [11], etc. The butterflies generated by these HT varieties are different from each other, which leads to the dependency of effective evaluations based on the butterfly. This however, also yields further difficulties.

This paper addresses the "ideal" HT data calculation, which is independent from the HT methods, digital discretisation errors in the image space, and parameters' errors due to the voting and verifying processes. The generated HT data is dependant only on the basic physical facts, the endpoints of the segment. The ideal HT data is employed to evaluate the data generated by the HT varieties. The ideal HT data may also be used to evaluate the method performance by measuring the margin of improvement of a specific segment detection method.

2. ERROR SOURCES OF HT

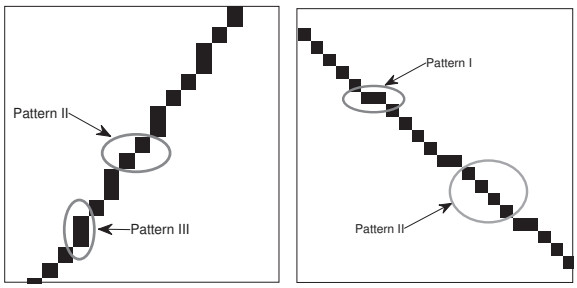
In HT, both the image space and the parameter space are discrete which introduces errors in the HT data. The image space discretisation errors are mainly manifested in the position aliases of feature points (i.e. the 1-pixels in the binary edging image), while the parameter space discretisation errors mainly affect the aliases between the peak position and the "true" parameters of the segment. These errors are the main error sources of HT. Ji et. al [12] discussed how the input errors (such as noise and pixel aliases) propagated to the detection error using Bayesian scheme. The following discussion will separate these errors and analyze how they affect the detection errors.

2.1 Discretisation Error in the Image Space

In digital imaging, when an image is captured the scene is divided to a lot of square areas. Each of these square areas is stored as a pixel in the image. This process introduces discretisation errors in the image. So for most digital image processing applications the discretization errors are unavoidable. In HT, these errors also seriously affect the quality of generated HT data.

Pixel Position Errors:

Commonly images are stored in the form of pixel matrix. Usually the pixels are considered as small squares connecting with 8 neighbors, with four pixel-pair possible directions (0°, 90°, and ±45°). A straight line is composed of pixel-pairs with arbitrary angles that are approximated with available ones (0°, 90°, and ±45°). This leads to aliases in pixel position, i.e. the pixels composing a straight line are not always collinear. As shown in Fig. 1 the straight line segments are composed of various 0° and ±45° pixel pairs when $-45° < \theta < 45°$ (Fig. 1(a)), while 90° and ±45° pixel pairs when $\theta > 45°$ or $\theta < -45°$ (Fig. 1(b)). The position errors due to this problem are common in almost all digital image algorithms and applications such as [13].



(a) Patterns for $-45° < \theta < 45°$ (b) Patterns for $\theta < -45°$ or $\theta > 45°$

Figure 1: 3 patterns for digital images

Pixel Numbers vs the Length of Segment:

It is ideal and straight if the number of pixels could present the length of the segment. However, this is not always true in digital images, since the pixels are considered as small squares only the length of vertical and horizonal segments can be directly and accurately presented by the number of their pixels. The number of pixels in all other segments is approximately equal to the length of their projection on horizonal direction (for $\theta > 45°$ or $\theta < -45°$) or vertical direction (for $-45° \leq \theta \leq 45°$). Unfortunately most HT methods, such as SHT, do not handle this problem. This is the main obstacle to HT being considered as a segment measurement tool.

2.2 Discretisation Error in the Parameter Space

In HT the parameter space is quantified to a matrix of cells or a similar structure. For instance, in SHT a matrix of cells presents the parameter space; the RHT parameter structure is a Binary Linked List where each node (similar to a cell in SHT) presents a straight line; FHT uses a k-tree to present the parameter space and each node (also similar to a cell in SHT). The size of a cell or node is the "resolution". A series of angles (or slopes) is predefined, when a feature point in the image space is concerned eq. (1) or its equivalent is used to calculate the other parameter (such as ρ or intercept) and vote to the nearest cells (or nodes). During this process, the predefined angles or slopes are accurate but the calculated values are rounded to the nearest quantified cells or nodes. This rounding error is the main source of parameter space error especially when the resolution is high (the size of a cell or node is very small). Of course the image space position errors also negatively affect the parameter space error. For example in Fig. 2, the cell corresponding to the solid line bounded image area will get votes from the feature points contained in the area. However, considerable votes are missed because of the feature point position aliases and rounding errors, which lead to low peaks even for long segments.

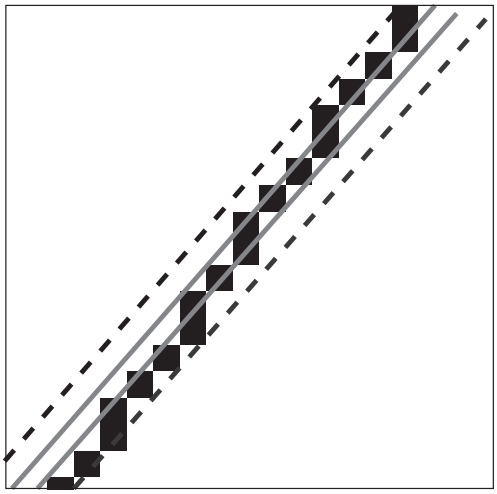


Figure 2: The discretisation error in the parameter space

3. IDEAL HT DATA INDEPENDENT OF HT ERRORS

SHT cannot be used either as a segment measuring tool or as a standard to evaluate other HT methods. The comparison of the HTs' performance via the number/accuracy of detected objects is not fair because of the lack of comparison of the intermediate data used (usually the HT data). Therefore, the method for assessing the performance of various HT methods is still an open problem. This section addresses the calculation of the ideal HT data as the standard to assess the HT data used in various HT methods, so as to obtain a better comparison.

3.1 Expectations on the ideal HT data

The ideal HT data should have the following features:

- HT data free from image space errors;
- HT data free from parameter space errors;
- The peak presents the length of the segment;
- HT data free from the effects of resolution settings;
- HT data measures the complete parameters of segments.

Considering these expectations and the fact that the ideal HT data is available, only when the parameters of the

segment are known, the calculation should only depend on the physical fact, i.e. the endpoints (x_1, y_1) and (x_2, y_2). So the peak can be determined as

$$\theta_p = \arctan \frac{y_2 - y_1}{x_2 - x_1}, \tag{2}$$

$$\rho_p = x_1 \cos\theta_p + y_1 \sin\theta_p, \tag{3}$$

and the height of the peak should be equal to the length of the segment, i.e.

$$H_p = \sqrt{(y_2 - y_1)^2 + (x_2 - x_1)^2}. \tag{4}$$

3.2 Calculation of the ideal HT data

To avoid rounding and quantification errors, the density of each cell is considered as the length of the intersection of its corresponding bar and the segment. The principle of calculating ideal HT data is shown in Fig. 3, where $\theta_0 = \theta_p$.

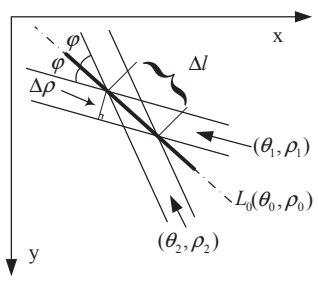


Figure 3: The principle of ideal HT data

A cell, for instance (θ_1, ρ_1), corresponding to the bar (θ_1, ρ_1) in the image space. The width of the bar is the ρ resolution $(\Delta\rho)$ and the angle between the bar and the segment is ϕ, so the length of their intersection when they fully intersect is

$$\Delta l = \frac{\Delta\rho}{\sin\phi}, \tag{5}$$

where

$$\phi = |\theta_0 - \theta_1|. \tag{6}$$

All bars parallel with the bar (θ_1, ρ_1) and fully intersect with the segment get the same length intersection, i.e. Δl. The bars partially intersect with the segment at its ends, denoted as (θ_1, ρ_{e1}) and (θ_1, ρ_{e2}) get shorter intersections:

$$\Delta l_{e1} = \frac{\rho^1_{\theta_1} - \rho_{e1}}{\sin\phi}, \tag{7}$$

$$\Delta l_{e2} = \frac{\rho_{e2} - \rho^2_{\theta_1}}{\sin\phi}, \tag{8}$$

where

$$\rho^1_{\theta_1} = x_1 \cos\theta_1 + y_1 \sin\theta_1, \tag{9}$$

$$\rho^2_{\theta_1} = x_2 \cos\theta_1 + y_2 \sin\theta_1, \tag{10}$$

are the theoretical distances from the origin to the straight lines containing the endpoints and having the angle equal to θ_1.

The following steps summarise the calculation of the ideal HT data.

- Input segment endpoints (x_1, y_1) and (x_2, y_2), and the ranges in θ and ρ directions of the parameter space where the HT data is to be calculated;
- Determine the peak in ideal HT data:
  - According to eqs. (2) and (3) calculate θ_p and ρ_p, i.e. the peak position;
  - Calculate the length of the segment by eq. (4), i.e. the height of the peak.
- Refine the given range according to the peak position; In SHT, the parameter space is scattered to a set of cells ranging over {-90°, $-90°+\Delta\theta$, ..., +90°; $-\rho_{max}$, $-\rho_{max}+\Delta\rho$, ..., $+\rho_{max}$}, where ρ_{max} is the maximum distance from the origin to a feature point in the image space. It is not reasonable to assume the existence of a cell in the parameter space exactly representing the ideal peak. So that the nearest cell to the ideal peak is moved to exactly represent the ideal peak and all other cells are moved accordingly.
- For each column around the peak, i.e. for each θ value in the refined range, for instance θ_1 in Fig. 3:
  - Calculate the first and the last cells in the parameter space whose bar partially intersects with the segment, according to eqs. (9) and (10);
  - Calculate the density of the first and the last cells according to eqs. (7) and (8);
  - Calculate the density of the middle cells whose bar in the image space fully intersects with the segment, according to eq. (5).

4. EVALUATE THE HT DATA GENERATED BY SPECIFIC ALGORITHM

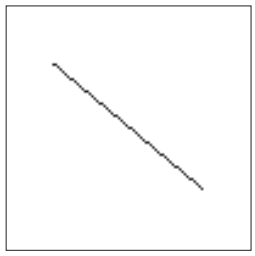
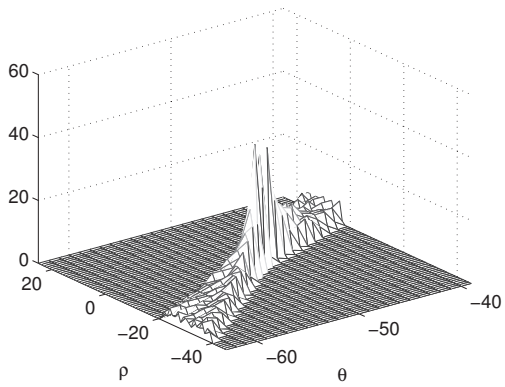
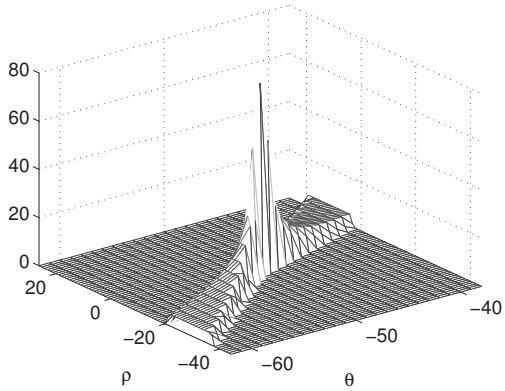


Figure 4: The segment used to evaluate SHT

In this section, an image containing a segment of 80 pixel lengths in Fig. 4 is given and the coordinates of the endpoints are known. As an example the performance of SHT is evaluated using the ideal HT data generated by the proposed method. The butterflies generated by SHT and the proposed method are shown in Figs. 5(a) and 5(b) respectively. It is easy to find out the differences of these two butterflies. Firstly, the peak height of SHT

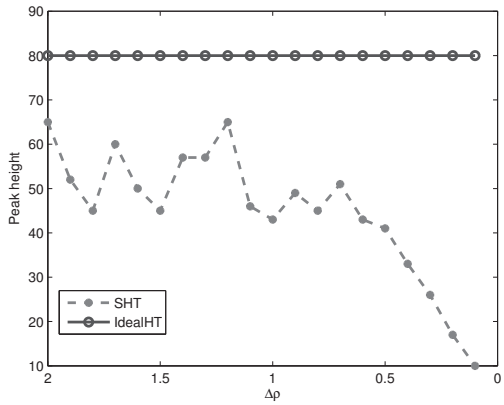


(a) Butterfly generated by SHT

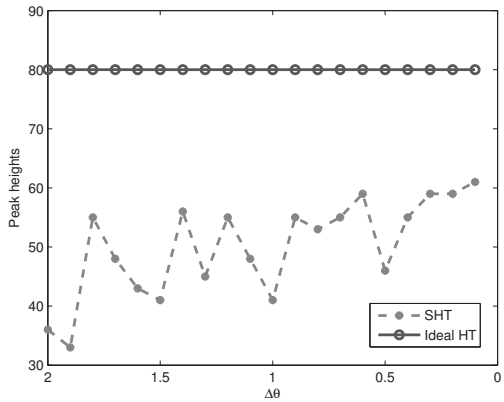


(b) Butterfly generated by the proposed method

Figure 5: Comparison on butterflies under the same resolutions ($\Delta\theta = 1^\circ, \Delta\rho = 1$pixel)



(a) The peak height vs $\Delta\rho$



(b) The peak height vs $\Delta\theta$

Figure 6: Peak height as the measurement of segment length

(about 60) is much smaller than the ideal HT data (about 80). Secondly, the HT data in each column (the HT data in ρ direction when θ value is specified) is undulate but the ideal HT data is very smooth. More quantitative comparisons are demonstrated as shown below.

4.1 The Peak Height vs the Segment Length

It is ideal if the peak height in HT data could present the length of the segment. However, in practice like other performances the peak height usually relates to the HT resolution. This will affect the detection errors as discussed by Tu et. al in [14].

For the given segment, firstly fix the θ resolution ($\Delta\theta = 1^\circ$) but change ρ resolution from 2 to 0.1 by 0.1 pixel each time ($\Delta\rho = 2, 1.9, ..., 0.1$ pixels). The peak heights detected by both SHT and the proposed method are depicted in Fig. 6(a). A strong decreasing trend is shown in the curve of SHT. This is because under high ρ resolution (very small $\Delta\rho$) the width of the bar corresponding to each cell is very small, many votes are missed due to both the feature point position aliases and the rounding errors during voting process. Of course, the peak heights of the ideal HT data keep the same value, because they are exactly equal to the length of the segment which is not affected by the HT resolutions.

Secondly, fix the ρ resolution ($\Delta\rho = 1^\circ$) but change θ resolution from 2 to 0.1 by 0.1 each time ($\Delta\theta = 2^\circ, 1.9^\circ, ..., 0.1^\circ$). The peak height generated by SHT shows a trend of approximating a maximum value (i.e. the number of pixels composing the segment). But never equal to the length of the segment unless the segment is vertical or horizonal.

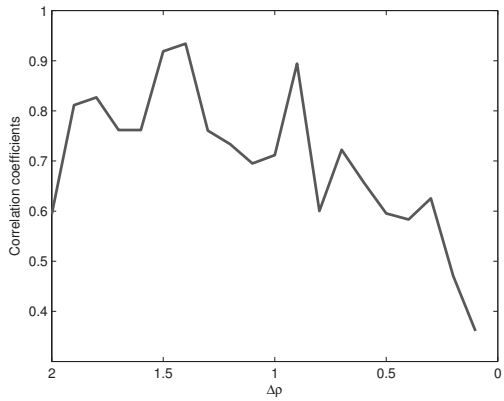
4.2 The Quality of the Generated Butterfly

For the butterfly based methods, the performance is strongly dependent on the quality of the HT butterfly. This experiment focuses on the quality of the butterflies generated by SHT. The correlation coefficient (defined as eq. (11)) between the ideal HT data and the one of SHT is used to measure the quality.

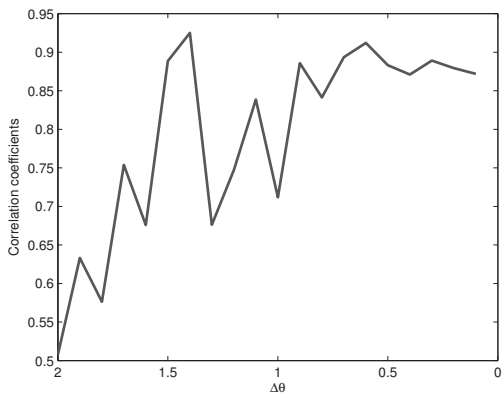
The correlation coefficient between A and B, where A and B are matrices or vectors of the same size, is defined as follows:

$$r(A,B) = \frac{\sum_m \sum_n (A_{mn} - \bar{A}) \times (B_{mn} - \bar{B})}{\sqrt{\left(\sum_m \sum_n (A_{mn} - \bar{A})^2\right) \times \left(\sum_m \sum_n (B_{mn} - \bar{B})^2\right)}}, \tag{11}$$

where $\bar{A}$ and $\bar{B}$ are the average value of matrix A and B respectively.



(a) The correlation vs Δρ



(b) The correlation vs Δθ

Figure 7: The quality of butterflies

Fig. 7 depicts the correlation coefficients when the resolutions changing. Clearly, increasing the ρ resolution (decrease Δρ) might lead to a poor quality of HT data. But increasing the θ resolution (decrease Δθ) shows a trend of increasing quality.

4.3 Effect of segment parameters

This section addresses the effect of segment parameters (i.e θ, ρ and length) on the performance demonstrated in sections 4.1 and 4.2.

Effect of the segment angle (θ) : To show the effect of θ, the segment depicted in Fig. 4 is rotated around the origin so the θ value was changed but the ρ value and the length were held. Then the experiments designed in sections 4.1 and 4.2 were repeated to get the results depicted in Fig. 8.

From the results in Fig. 8, one can find that the θ value does not have considerable effect on the peak height of the butterfly and its quality.

Effect of the distance from the origin to the segment (ρ): To check the effect of ρ, the segment depicted in Fig. 4 is moved horizontally so the ρ value was changed but the θ value and the length were held. The results are shown in Fig. 9.

From the results in Fig. 9, it may be also noticed that the value of ρ does not have considerable effect on the peak height of the butterfly and its quality.

Effect of segment length:

Now hold both θ and ρ but just change the segment length, i.e. hold one endpoint but move the other one along the segment. The result is shown in Fig. 10. From Figs. 10(b) and 10(a) one still cannot find considerable effect. But from Figs. 10(c) and 10(d) one can get that the peak height for shorter segment is closer to the ideal value. This is because, for the shorter segment, it is easier for the bar corresponding to the peak cell to contain a bigger percentage of pixels composing the segment. On the contrary, longer segment has more chance to have a part lying outside of the narrow bar corresponding to the peak cell.

Of course, the shorter segments get more chances of being affected by noise and other objects.

4.4 Evaluation of the SHT

From Sections 4.1 and 4.2 the following conclusions on the SHT are depicted:

- The HT data generated by the SHT is greatly affected by the resolution;
- The HT data generated by the SHT under the best resolution is still far from the ideal HT data;
- Higher ρ resolution (i.e smaller Δρ) tends to negatively affect the HT data quality. However, higher θ resolution (i.e smaller Δθ) tends to give positive effects.

5. WEAKNESS OF THE PROPOSED IDEAL HT

The proposed ideal HT was designed to evaluate complete HT butterflies, i.e. all feature points are considered when generating HT data. However, such complete butterflies are not available for some HT varieties generating HT data considering a subset of feature points only, such as the randomised HT [10] where feature point pairs are randomly selected and hence the generated butterfly is incomplete.

6. CONCLUSION

This paper proposed a method for calculating the ideal HT data of segments. The HT data thus generated is not affected by the errors from the image space position aliases or the parameter space rounding errors. It only depends on the basic physical factors (the endpoints of the segment).

The ideal HT data can be used to evaluate the performance of various HTs. The SHT is evaluated as an example.

If an appropriate range in the parameter space is given, the generated butterfly is potentially used to detect segments because it can be considered as a model of a segment. Furthermore, the proposed method is hopefully used to optimise the resolutions of specific HT varieties to obtain the "best" performance. This will be focused on in future work.

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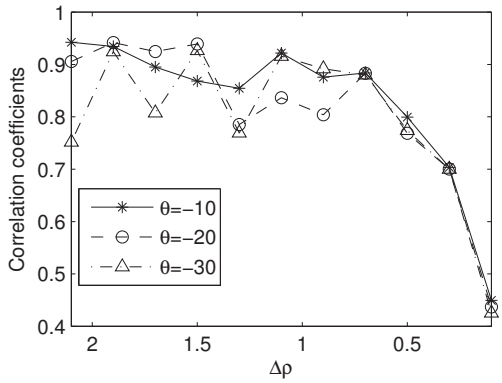
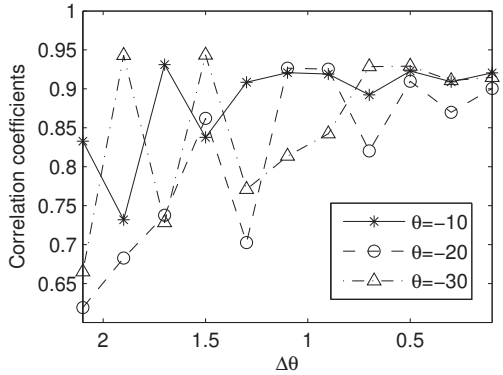
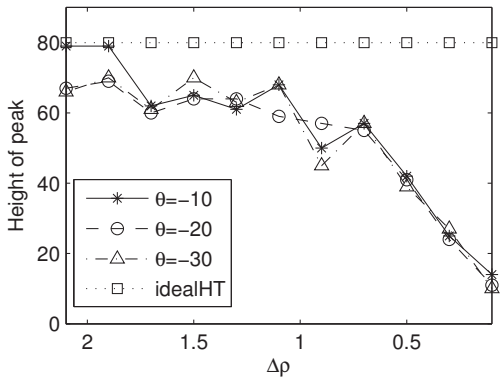


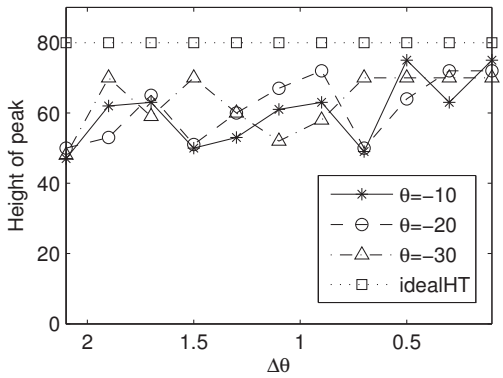
(a) The correlation vs Δρ for segments with different θ values



(b) The correlation vs Δθ for segments with different θ values

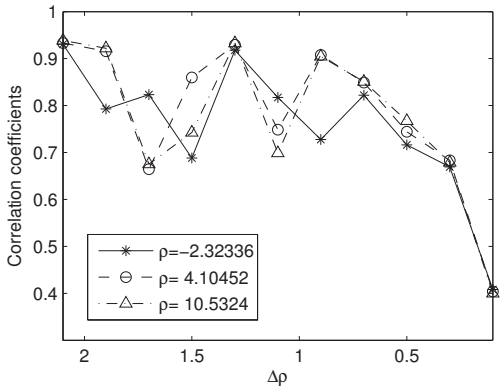


(c) The peak height vs Δρ for segments with different θ values

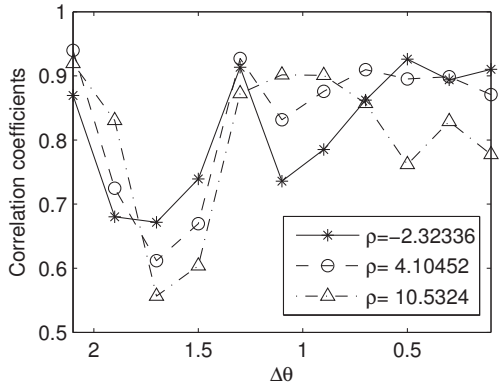


(d) The peak height vs Δθ for segments with different θ values

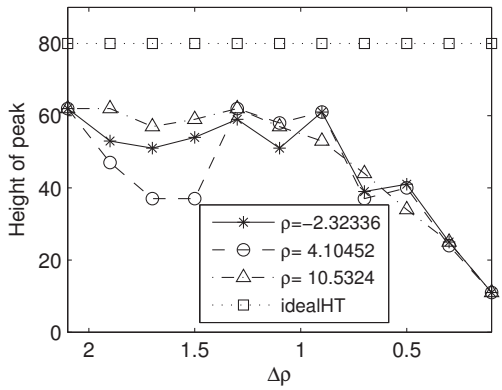
Figure 8: The effect of segment θ value



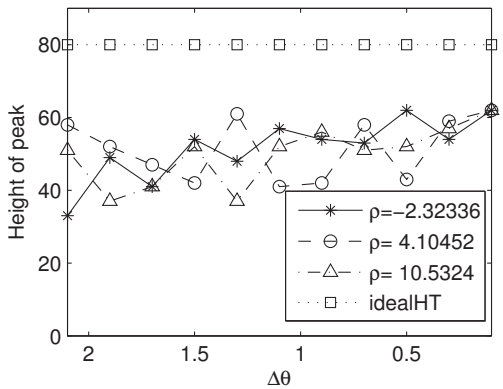
(a) The correlation vs Δρ for segments with different ρ values



(b) The correlation vs Δθ for segments with different ρ values

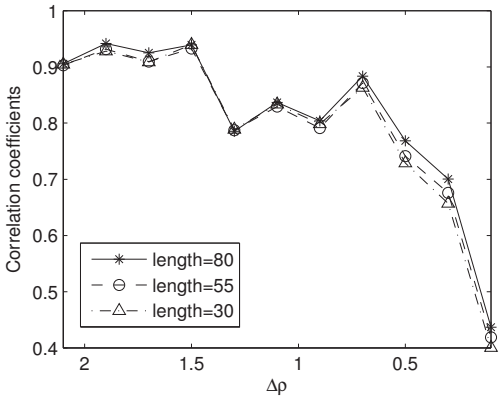


(c) The peak height vs Δρ for segments with different ρ values

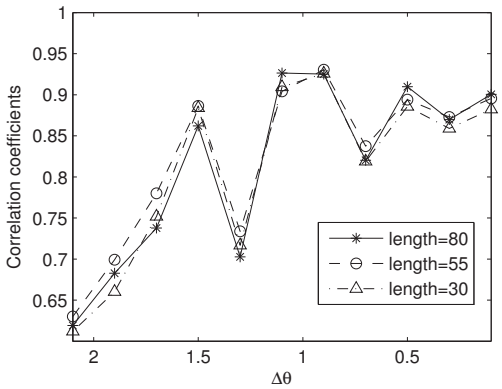


(d) The peak height vs Δθ for segments with different ρ values

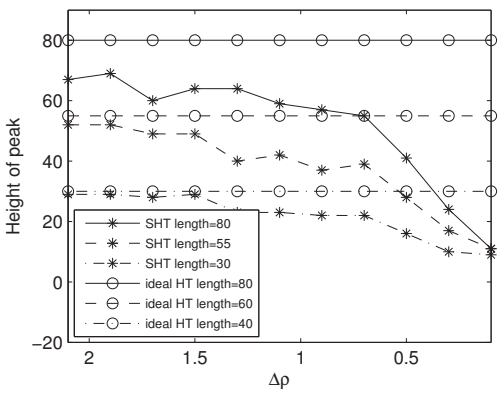
Figure 9: The effect of segment ρ value



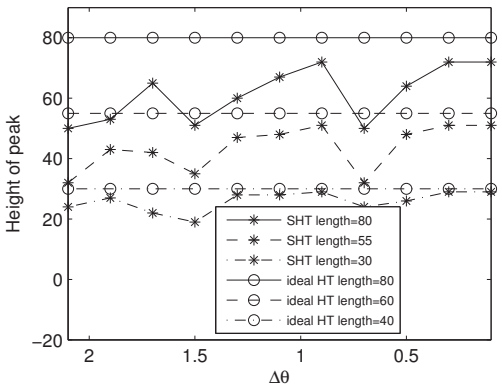
(a) The correlation vs Δρ for segments with different length



(b) The correlation vs Δθ for segments with different length



(c) The peak height vs Δρ for segments with different length



(d) The peak height vs Δθ for segments with different length

Figure 10: The effect of segment length value