PERFORMANCE ANALYSIS AND SIMPLIFIED DETECTION FOR TWO-DIMENSIONAL SIGNAL SPACE DIVERSITY WITH MRC RECEPTION

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Abstract: Signal space diversity (SSD) is a promising technique for obtaining diversity without increases in bandwidth, transmit power or physical hardware at the expense of increased ML detection complexity. Symbol error rate (SER) analysis of a two-dimensional system containing a single transmit antenna and N receive antennas with maximal-ratio combining (MRC) reception is presented here along with a simplified detection scheme for SSD systems. The union bound and the nearest neighbour (NN) approximation are presented in closed form, and a new, simpler SER bound for SSD systems based on the minimum Euclidean distance of a rotated constellation is presented, also in closed form. Performance of the new bound is found to be tight for low signal-to-noise ratios (SNRs), small rotation angles and when the number of receive antennas (N) is large; the new bound is also easily applied to other systems. The simplified detection scheme, while losing diversity and SER performance when $N = 1$, achieves a 7dB and 4dB performance improvement over non-SSD transmission at SERs of 10^{-3} and 10^{-2} for 4-QAM and 16-QAM respectively. However, when $N \geq 3$, SER performance is close to indistinguishable from that of optimal ML detection while achieving complexity reductions of up to 5.77%, 70.19% and 91.77% for 4-QAM, 16-QAM and 64-QAM respectively.

Key words: signal space diversity, lower bound, maximum-likelihood, maximal-ratio combining

1. INTRODUCTION

Diversity schemes typically rely on multiple copies of a transmitted signal arriving at the receiver over independent channels, which may be orthogonal in time or frequency or separated in space. These redundant copies available at the destination may then be combined in various ways [1] depending on the processing and hardware constraints at the receiver. Typically, spatial diversity schemes use multiple transmit and/or receive antennas [2] to generate the redundant signal copies. Spatial diversity, while capable of providing substantial gains, requires additional antennas and enough distance between them to avoid correlation at either the transmitter or receiver and hence is not a practical solution in all cases.

A diversity technique which has not seen much attention is signal space diversity (SSD) [3]. SSD exploits the inherent diversity available in the different dimensions of a multi-dimensional constellation by ensuring that different components are each affected by independent fading. Consequently, no additional bandwidth, transmit power or space is required, however this is achieved at the cost of necessitating a more complicated maximum-likelihood (ML) detector at the receiver. This makes SSD a useful technique in the modern mobile communication focused age, as more processing power becomes available in mobile devices but space constraints are not lifted. Second generation digital terrestrial television systems already employ rotated constellations to provide error performance and diversity gains [4].

A multi-dimensional constellation is said to be capable of L order diversity when the minimum number of unique components between any two points of the constellation is L [3]; this property gives a two-dimensional constellation diversity of order two. Error performance is further improved by maximising the minimum product distance of a constellation [3]. The maximum minimum product distance is realised by optimum selection of the rotation angle, derived for SISO systems in [5].

It was demonstrated in [5] that the union bound could be used to evaluate error performance of rotated constellations with component interleaving by summing over the pairwise error probabilities (PEP) between any arbitrary constellation point and every other constellation point. The pairwise error probability has been found in closed form in [5] and [6] for M-QAM and PSK modulations respectively, and is sometimes used in conjunction with the Chernoff bound [5]. The union bound, while asymptotically tight, is loose at low signal-to-noise (SNR) and for larger constellations [7], while the Chernoff bound is looser by close to 4dB [5].

The nearest neighbour (NN) approximation approximates the union bound by only considering the PEP of the nearest neighbours of any constellation point. The NN approximation was presented in [8] for MRC reception and M-QAM modulation, and in [6] for PSK modulation.

However, no closed form solution was presented for M-QAM with MRC reception in [8].

Recently, exact error expressions have been derived for SSD systems by introducing a change in signal model and by using polar coordinates for both Rayleigh [9] and Rician [10] channels. The new signal model takes the ratio of the standard deviation of the in-phase and quadrature components into account, and hence states that the resulting decision regions become non-perpendicular. The conditional probability of error is then integrated over these non-perpendicular regions defined by the angles between them using polar coordinates. The derived expressions, while accurate, are not presented in closed form and require numerical evaluation of integrals.

Also recently, a lower bound for uncoded rotated lattice constellations based on the sphere lower bound (SLB) was proposed in [11] for Nakagami-m fading. The SLB shown in [11] is based on the integration of the Voronoi region shown in [7]. It was shown that the SLB exhibits good performance for infinite lattices regardless of the lattice structure. However for finite multi-dimensional constellations obtained from the rotation of M-PAM (Pulse Amplitude Modulation) constellations, the SLB only exhibits tight performance as M gets large.

An accurate and simple to compute closed form expression for the error probability of rotated multi-dimensional constellations with SSD does not yet exist. In this study, we attempt to derive such an expression in the form of a lower bound based on the minimum Euclidean distance of a rotated constellation. The lower bound is presented in closed form and compared with the union bound and NN approximations, which are also presented here in closed form, for a system containing a single transmit antenna and multiple receive antennas with MRC reception.

The optimal ML detection rule requires an exhaustive search among all constellation points before estimating the transmitted symbol. Therefore, the possibility of reducing the complexity of ML detection exists. A simplified detection scheme is presented here which searches m constellation points from a total of M after performing initial signal conditioning. The performance of the simplified detection scheme is compared to that of the optimal detection scheme using simulation for both single and multiple antenna reception.

The rest of this paper is organized as follows: Section 2 presents the system model, Section 3 presents performance analysis of the various bounds, Section 4 presents the simplified detection scheme, results are presented in Section 5 and Section 6 presents concluding remarks.

2. SYSTEM MODEL

Consider the conventional (S) and rotated ($\tilde{S}$) constellations shown in Figure 1. Clearly, rotating the constellation maximizes the minimum number of distinct components between any two points in the constellation while retaining the same average energy, given by the expectation $E[S^2]$, where $E[\cdot]$ denotes the expectation operator.

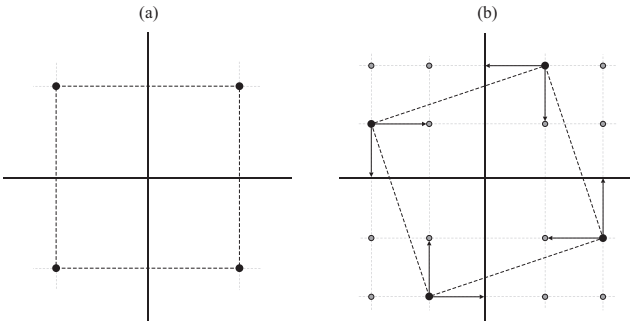

Figure 1: (a) original constellation S and (b) rotated constellation $\tilde{S}$ showing expansion

We formally define S to be a signal set such that $S = \left\{ s_l^I + j s_l^Q : l = 0, 1, ..., M-1 \right\}$, where M denotes the cardinality of S, and $(\cdot)^I$, $(\cdot)^Q$ refer to the in-phase and quadrature part of a signal respectively. The rotated set $\tilde{S}$ is then obtained by applying a rotation matrix $\mathbf{R}^\theta$ to each element of S according to $\tilde{s}_l = \mathbf{R}^\theta s_l$, for a rotation of angle θ. For a two-dimensional constellation, the rotation matrix takes the form [12]:

$$\mathbf{R}^\theta = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}. \tag{1}$$

To obtain any diversity, the unique components need to be affected by independent fading. This is achieved by a component interleaver/de-interleaver present at the transmitter and receiver respectively. Performing the interleaving/de-interleaving action ensures that a single deep fade will not affect all components of the signal simultaneously, thus achieving signal space diversity. Note that the rotated constellation, once component interleaved, now forms an expanded constellation equivalent to the Cartesian product of the in-phase and quadrature components of the original constellation set, defined as $\tilde{S}_E = S^I \times S^Q$, where $\times$ denotes the Cartesian product. This can be visualized in Figure 1.

We now consider the system with block diagram shown in Figure 2. Consider a network containing a source node with a single transmit antenna and a destination node containing N receiving antennas, also referred to here as N independent branches. The receive antennas are spaced far enough apart for there to be no correlation among the respective received signals. There is no feedback channel between the destination and source nodes and no source or channel coding is performed.

Following the system block diagram, a bit stream is

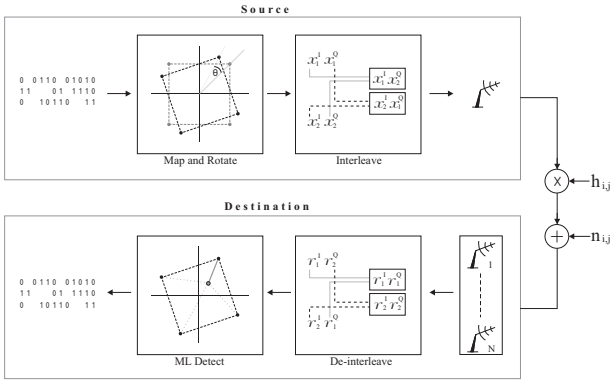

Figure 2: Block diagram of SSD system with MRC reception

mapped to symbols denoted $x = x^I + jx^Q$ from the rotated constellation $\tilde{S}$ as shown in the first block in Figure 2, which are then arbitrarily grouped into pairs of symbols and passed through a component interleaver, shown in the second block. The component interleaver interleaves the in-phase and quadrature components of the symbols in each pair to give new symbols u_t, $t \in \{1,2\}$, where t is the index of a symbol in a symbol pair. A typical interleaved symbol pair is shown below:

$$\begin{aligned} u_1 &= x_1^I + jx_2^Q \\ u_2 &= x_2^I + jx_1^Q. \end{aligned} \tag{2}$$

The interleaved symbols u_t are then transmitted by the single transmit antenna and arrive at the destination over N i.i.d (independent and identically distributed) Rayleigh fading channels with additive white Gaussian noise from the N receive antennas. Each symbol u_t in a symbol pair is transmitted over an orthogonal channel, assumed to be independent time slots in this study. Transmission occurs in two subsequent time slots, with u_1 transmitted in the first time slot and u_2 in the second. Therefore, the SSD system consumes no more bandwidth than comparable non-SSD systems. Thus we denote the received symbols to be $r_{i,j}$, where the subscript $i \in \{1,2\}$ is the orthogonal channel (time slot) index and the subscript $j \in \{1,2,...,N\}$ is the receive antenna index. The received symbols at antenna j are then given by:

$$r_{i,j} = h_{i,j}u_t + n_{i,j} \tag{3}$$

where $h_{i,j}$ is the fading coefficient in time slot i at antenna j. We assume that full channel state information (CSI) of all paths is available at the receiver and that no inter-symbol interference occurs, hence the receiver is able to remove the phase shift induced by fading. Thus the fading is modelled as independent Rayleigh distributed random variables with amplitude distributed according to $f_h(h_{i,j}) = \frac{h_{i,j}}{\sigma^2}\exp\left(\frac{-h_{i,j}^2}{2\sigma^2}\right)$ and unit second moment, i.e. $E\left[h_{i,j}^2\right] = 2\sigma^2 = 1$. The fading is assumed to be flat. The transmitted signals are perturbed by additive white Gaussian noise, modelled as circular symmetric Gaussian random variables with distribution $n_i \sim CN(0,N_0)$, i.e. zero mean and variance $N_0/2$ per dimension.

Diversity combining is performed at the destination in the optimal fashion using MRC. Symbols are combined on a symbol-by-symbol basis as they are received, however detection is only performed after a symbol pair from all N antennas has been received, combined and de-interleaved. Combining is performed by weighting the received signal from each path with the respective instantaneous fading coefficient for that path. This gives the combined signal for time slot i:

$$r_i = h_{i,1}r_{i,1} + h_{i,2}r_{i,2} + ... + h_{i,N}r_{i,N} \tag{4}$$

where we have defined the combined signal to be r_i. This is similar to combining in a non-SSD system, and combines the branches in the optimal fashion. We further define an instantaneous combined fading term based on the sum of the individual instantaneous branch fading powers:

$$h_i^2 = h_{i,1}^2 + h_{i,2}^2 + ... + h_{i,N}^2. \tag{5}$$

De-interleaving is then performed on a symbol pair to ensure that the in-phase and quadrature components of the original symbol are reassembled before detection. The symbols are then detected in the ML sense after de-interleaving, assuming full CSI. For MRC reception, the ML rule can be written as, for both time slots [13, 14]:

$$\begin{aligned} \hat{x}_1 &= \arg\min_{x_k \in \tilde{S}} \left\{ h_2^2 \left|r_1^I - h_1^2 x_k^I\right|^2 + h_1^2 \left|r_1^Q - h_2^2 x_k^Q\right|^2 \right\} \\ \hat{x}_2 &= \arg\min_{x_k \in \tilde{S}} \left\{ h_1^2 \left|r_2^I - h_2^2 x_k^I\right|^2 + h_2^2 \left|r_2^Q - h_1^2 x_k^Q\right|^2 \right\} \end{aligned} \tag{6}$$

where h_i^2 has been defined in (5), and $\hat{x}_i$ are the detected symbols for time slot i. Note the addition of the combined fading coefficient in each term of the ML decision rule, e.g. the h_2^2 in $h_2^2\left|r_1^I - h_1^2 x_k^I\right|^2$.

3. PERFORMANCE ANALYSIS

In this section we first review the conventional approach to approximating the error probability of SSD systems based on the union bound as presented in [3, 5, 6]. We then review the NN approximation [8] and present it in closed form, since the NN approximation often has tighter error performance, especially at low SNR. We thereafter present a simpler minimum Euclidean distance based lower bound which is applicable to any square M-QAM constellation.

For convenient discussion, we use $f(x)$ to denote the PDF of random variable x, $F(x)$ to denote its CDF, and $P(z)$ to denote the probability of event z.

3.1 Union Bound

The union bound gives a standard method of evaluating the error probability of an arbitrary signal set by summing the PEPs across all possible transmit and receive pairs. The union bound P_S^U on the symbol error probability (SER) is given as [5]:

$$P_S^U(e) \leq \frac{1}{|\tilde{S}|} \sum_{x \in \tilde{S}} \sum_{\substack{\hat{x} \in \tilde{S} \\ x \neq \hat{x}}} P(x \to \hat{x}) \tag{7}$$

where $|\tilde{S}|$ denotes the cardinality of the signal set and $P(x \to \hat{x})$ is the unconditional PEP of the detector choosing $\hat{x}$ given that x was transmitted. Letting $\gamma_i = h_i^2 (E_S/N_0)$ be the combined instantaneous SNR in time slot i, the conditional PEP follows from the ML rule expressed in equation (6) and can be written as [5]:

$$\begin{aligned} P(x \to \hat{x}|h_1,h_2) &= P\left(\|\mathbf{r} - \mathbf{h} \odot \mathbf{x}\|^2 \geq \|\mathbf{r} - \mathbf{h} \odot \hat{\mathbf{x}}\|^2 |h_1,h_2\right) \\ &= P\left(\|\mathbf{n}\|^2 \geq \|\mathbf{r} - \mathbf{h} \odot \hat{\mathbf{x}}\|^2 |h_1,h_2\right) \\ &= Q\left(\sqrt{\frac{1}{2N_0}\left(h_1^2 d_1^2 + h_2^2 d_2^2\right)}\right) \\ &= Q\left(\sqrt{\frac{1}{2E_S}\left(\gamma_1^2 d_1^2 + \gamma_2^2 d_2^2\right)}\right) \end{aligned} \tag{8}$$

where $\mathbf{r} = \left(r^I, r^Q\right)$, $\mathbf{h} = (h_1, h_2)$, $\mathbf{x} = \left(x^I, x^Q\right)$ and $\hat{\mathbf{x}} = \left(\hat{x}^I, \hat{x}^Q\right)$ are the two-dimensional received signal, channel coefficient, transmitted and chosen signal vectors respectively, the $\|\cdot\|$ operation represents the vector norm, the $\odot$ operation represents the vector product, E_S is the average energy per symbol, and d_1^2, d_2^2 represent the in-phase and quadrature distances between x and $\hat{x}$ respectively. Averaging the conditional PEP over the independent fading distributions for MRC reception gives the unconditional PEP, where $*$ denotes multiplication:

$$\begin{aligned} P(x \to \hat{x}) = \int_0^\infty \int_0^\infty Q\left(\sqrt{\frac{1}{2E_S}\left(\gamma_1^2 d_1^2 + \gamma_2^2 d_2^2\right)}\right) * \\ f_{\gamma_{MRC}}(\gamma_1) f_{\gamma_{MRC}}(\gamma_2)\, d\gamma_1 d\gamma_2. \end{aligned} \tag{9}$$

Closed form expressions for the PEP have been shown in [5] and [6] for M-QAM and PSK modulations respectively in Rayleigh fading with a single transmit/receive antenna. Substituting (9) into (7) yields the final union bound on the error probability.

3.2 Nearest Neighbour Approximation

The NN approximation can be used to simplify the union bound by considering only the points closest in Euclidean distance to any symbol in a constellation [6, 8]. The NN approximation was presented in [8] for a system employing MRC reception, however no closed form expressions were presented. We thus present closed form expressions for the NN approximation with N-branch receive MRC using the trapezoidal approximation to the $Q(\cdot)$ function, presented later in this section.

Following the approach presented in [8], the NN approximation for 4-QAM is equivalent to the union bound since the nearest points constitute the entire constellation [6]. For 16-QAM, the NN approximation can be written, by considering the points in the corners, centre, and sides of the constellation, as [8]:

$$P_{16-QAM}^{NN}(e) = \frac{1}{4} P_{corner}(e) + \frac{1}{4} P_{centre}(e) + \frac{1}{2} P_{side}(e) \tag{10}$$

where $P_{corner}(e)$, $P_{centre}(e)$, and $P_{side}(e)$ are the error probabilities of the points located at the corners, centre and sides of the constellation respectively. Each of the described points has perpendicular and diagonal neighbours with different Euclidean distances. By considering the different neighbours, the NN approximation for 16-QAM can be written as:*

$$\begin{aligned} P_{16-QAM}^{NN}(e) = &\frac{1}{4}\left[2P_{perp}(x \to \hat{x}) + P_{diag}(x \to \hat{x})\right] + \\ &\frac{1}{4}\left[4P_{perp}(x \to \hat{x}) + 4P_{diag}(x \to \hat{x})\right] + \\ &\frac{1}{2}\left[3P_{perp}(x \to \hat{x}) + 2P_{diag}(x \to \hat{x})\right] \\ = &\, 3P_{perp}(x \to \hat{x}) + \frac{9}{4} P_{diag}(x \to \hat{x}) \end{aligned} \tag{11}$$

where $P_{perp}(x \to \hat{x})$ and $P_{diag}(x \to \hat{x})$ are the PEPs between any point and its perpendicular and diagonal neighbours respectively. The PEPs can be found directly by substituting the relevant distances into (9). These distances are easily computed for perpendicular and diagonal neighbours as:**

$$\begin{aligned} d_{1_{perp}}^2 &= 4\cos^2\theta & d_{2_{perp}}^2 &= 4\sin^2\theta \\ d_{1_{diag}}^2 &= 4(1+\sin 2\theta) & d_{2_{diag}}^2 &= 4(1-\sin 2\theta). \end{aligned} \tag{12}$$

After making the relevant substitutions, integrating over the fading distributions gives the final PEP. We make use of the trapezoidal approximation to the $Q(x)$ function shown in [15] to simplify analysis. The $Q(\cdot)$ function can be approximated as:

* Equation (21) in [13] incorrectly calculates the number of diagonal neighbours for $P_{centre}(e)$ and $P_{side}(e)$, the corrected calculation is presented here.

** The diagonal distance is reported incorrectly in [13]. The corrected distance is shown here.

$$Q(\delta) = \frac{1}{2n}\left(\frac{1}{2}\exp\left(\frac{-\delta^2}{2}\right) + \sum_{c=1}^{n-1}\exp\left(\frac{-\delta^2}{S_c}\right)\right) \quad (13)$$

where $S_c = 2\sin^2(c\pi/2n)$ and n is the total number of iterations used in the approximation. The PEP can thereafter be written using (9), (12) and (13) for N-branch MRC reception as:

$$\begin{aligned} &P(x \to \hat{x}) \\ &= \int_0^\infty \int_0^\infty \left[\frac{1}{2n}\left(\frac{1}{2}\exp\left(\frac{-\chi}{2}\right) + \sum_{i=1}^{n-1}\exp\left(\frac{-\chi}{S_i}\right)\right)\right] * \\ &\quad f_{\gamma_{MRC}}(\gamma_1) f_{\gamma_{MRC}}(\gamma_2)\, d\gamma_1 d\gamma_2 \\ &= \frac{1}{4n}\left(\frac{2}{\rho_{1_z}^2\bar{\gamma}+2}\right)^N \left(\frac{2}{\rho_{2_z}^2\bar{\gamma}+2}\right)^N + \\ &\quad \frac{1}{2n}\sum_{i=1}^{n-1}\left(\frac{S_i}{\rho_{1_z}^2\bar{\gamma}+S_i}\right)^N \left(\frac{S_i}{\rho_{2_z}^2\bar{\gamma}+S_i}\right)^N \end{aligned} \quad (14)$$

where $\chi = \frac{1}{2E_S}\left(\gamma_1^2 d_1^2 + \gamma_2^2 d_2^2\right)$, $f_{\gamma_{MRC}}(\gamma_i) = \frac{1}{(N-1)!}\frac{1}{\bar{\gamma}^N}\gamma_i^{N-1}\exp\left(\frac{-\gamma_i}{\bar{\gamma}}\right)$ is the well-known MRC PDF for N receive antennas, $\rho_{i_z}^2 = \left(d_{i_z}^2/2E_S\right)$ and z refers to the *diag* or *perp* distances in (12). Substituting the relevant distances from (12) into (14) and thereafter (14) into (11) gives the final NN approximation to the union bound for N-branch MRC reception for 16-QAM. The expression in (14) can also be used in the union bound shown in (7) after computing and substituting the relevant distances.

3.3 Minimum Distance Lower Bound

The union bound sums the PEPs between any transmitted symbol and all possible received symbols to upper bound the SER, making use of the squared Euclidean distance between the points under consideration. We now derive a lower bound on the SER using the minimum Euclidean distance of the constellation, taking into consideration the independent fading on the individual components of the signal set.

The minimum distance of a constellation is not affected by rotation. However, in a system employing SSD, the individual components of the signal are affected by independent fading due to the interleaving and de-interleaving action. We therefore modify the standard minimum distance calculation, $d_{min} = \min_{x_k, x_l \in S}\left\{\sqrt{|x_k - x_l|^2}\right\}$, to account for the fading on the individual components of the signal:

$$d_{min} = \min_{x_k, x_l \in \tilde{S}}\left\{\sqrt{h_1^2\left|x_k^I - x_l^I\right|^2 + h_2^2\left|x_k^Q - x_l^Q\right|^2}\right\} \quad (15)$$

where h_1 and h_2 refer to the independent fading affecting the in-phase and quadrature parts of the signal from the first and second time slot respectively. This in effect is the minimum distance of the constellation after the rotation, interleaving, transmission over fading channel, and de-interleaving operations. Now, due to the independent fading on the individual dimensions, the minimum distance is dependent on the rotation angle.

Assuming that an M-QAM constellation has a spacing of two in each dimension between neighbouring constellation points, the minimum distance for a rotated two-dimensional constellation $\tilde{S}$ can be computed as:

$$d_{min}^{\tilde{S}} = \sqrt{4h_1^2\cos^2\theta + 4h_2^2\sin^2\theta}. \quad (16)$$

The conventional square M-QAM SER conditioned upon the instantaneous SNR can be written as [17]:

$$P_S^{M-QAM}(e|\gamma) = 4aQ\left(\sqrt{b\gamma}\right) - 4a^2Q^2\left(\sqrt{b\gamma}\right) \quad (17)$$

where $a = \left(1 - \frac{1}{\sqrt{M}}\right)$ and $b = \frac{3}{M-1}$. We again make use of the trapezoidal approximation to the $Q(x)$ and $Q^2(x)$ functions shown in [15] to simplify analysis. The conditional M-QAM SER for any square M-QAM constellation can be written using the mentioned approximations as:

$$\begin{aligned} &P_S^{M-QAM}(e|\gamma) \\ &= 4aQ\left(\sqrt{b\gamma}\right) - 4a^2Q^2\left(\sqrt{b\gamma}\right) \\ &= \frac{a}{n}\left\{\begin{array}{l} \frac{1}{2}\exp\left(\frac{-b\gamma}{2}\right) - \frac{a}{2}\exp(-b\gamma) + \\ (1-a)\sum_{c=1}^{n-1}\exp\left(\frac{-b\gamma}{S_c}\right) + \sum_{c=n}^{2n-1}\exp\left(\frac{-b\gamma}{S_c}\right) \end{array}\right\} \end{aligned} \quad (18)$$

where $S_c = 2\sin^2 c\pi/4n$. The factor $\sqrt{d_{min}^2/2N_0}$ can be substituted into the conditional SER in (18) (similar to the factor $\sqrt{d_i^2/2N_0}$ in the PEP in (8)) in place of $\sqrt{b\gamma}$ to give (19). Doing so lower bounds the SER since not all error events are taken into consideration. We henceforth term the new lower bound the minimum distance lower bound (MDLB).

$$P_S^{MDLB}(e|h_1, h_2) \geq 4aQ\left(\sqrt{\frac{d_{min}^2}{2N_0}}\right) - 4a^2Q^2\left(\sqrt{\frac{d_{min}^2}{2N_0}}\right). \quad (19)$$

After some simple manipulations, we arrive at the final conditional MDLB on the SER in equation (20), with

the terms a and b defined as they were in (17), and defining $\gamma_i = h_i^2 (E_S/N_0)$ to be the combined instantaneous SNR from the N receive antennas. Notice now that the final conditional MDLB resembles the exact conditional M-QAM SER with the instantaneous SNR changed to represent the independent fading on each dimension and the SSD rotation angle:

$$P_S^{MDLB}(e|\gamma_1,\gamma_2) \geq 4aQ\left(\sqrt{b\zeta}\right) - 4a^2Q^2\left(\sqrt{b\zeta}\right) \quad (20)$$

where $\zeta = \left(\gamma_1 \cos^2\theta + \gamma_2 \sin^2\theta\right)$. The conditional MDLB is then averaged over both fading distributions to give the final bound on the error probability:

$$P_S^{MDLB}(e) \geq \int_0^\infty \int_0^\infty \left\{4aQ\left(\sqrt{b\zeta}\right) - 4a^2Q^2\left(\sqrt{b\zeta}\right)\right\} * f_{\gamma_{MRC}}(\gamma_1) f_{\gamma_{MRC}}(\gamma_2)\, d\gamma_1 d\gamma_2. \quad (21)$$

Substituting the relevant fading distributions for MRC with N receive antennas and then performing the double integration gives the final expression for the unconditional MDLB:

$$P_S^{MDLB}(e) \geq \frac{a}{n}\left\{\begin{array}{l} \frac{1}{2}\left(\frac{2}{\alpha b\overline{\gamma}+2}\right)^N\left(\frac{2}{\beta b\overline{\gamma}+2}\right)^N - \\ \frac{a}{2}\left(\frac{1}{\alpha b\overline{\gamma}+1}\right)^N\left(\frac{1}{\beta b\overline{\gamma}+1}\right)^N + \\ (1-a)\sum_{c=1}^{n-1}\left(\frac{S_c}{\alpha b\overline{\gamma}+S_c}\right)^N\left(\frac{S_c}{\beta b\overline{\gamma}+S_c}\right)^N + \\ \sum_{c=n}^{2n-1}\left(\frac{S_c}{\alpha b\overline{\gamma}+S_c}\right)^N\left(\frac{S_c}{\beta b\overline{\gamma}+S_c}\right)^N \end{array}\right\} \quad (22)$$

where $\alpha = \cos^2\theta$, $\beta = \sin^2\theta$ and $\overline{\gamma} = E[\gamma]$. The new MDLB can be easily computed, is free from any complicated mathematical functions due to the approximations used, is presented in closed form, and can easily be applied to any system already using the conditional square M-QAM SER expression from (17) by making a simple substitution.

4. SIMPLIFIED DETECTION

The conventional ML detection rule in (6) performs an exhaustive search among all points in a constellation to find the one closest in squared Euclidean distance to the received symbol using the ML decision criterion. While providing optimal error performance, this is far from efficient. We thus propose a sub-optimal simplified detection scheme that first equalises the received symbols and then, after de-interleaving, selects the point closest in Euclidean distance to the equalised symbol. A search using the ML decision criterion is then performed among the m points closest to the equalised point, where $1 \leq m \leq M$. The new ML detection rule thus significantly reduces the complexity of ML detection in terms of the number of searches required.

The simplified detection scheme is split into two phases: the equalisation and selection phase, and the search phase. As with conventional ML detection, symbols can only be detected once both symbols in a symbol pair have been received and de-interleaved. The equalisation and selection phase first equalises the symbols received in each time slot by dividing by the known fading coefficient for that time slot to give symbols y_i. A typical equalised symbol pair is shown below, where the equalisation is performed knowing that h_i (defined in (5)) is an amplitude distribution:

$$\begin{aligned} y_1 &= \frac{r_1}{h_1} = u_1 + \frac{n_1}{h_1} \\ y_2 &= \frac{r_2}{h_2} = u_2 + \frac{n_2}{h_2} \end{aligned} \quad (23)$$

Note that the symbols u_i are still component interleaved; they must now be de-interleaved to reassemble the respective in-phase and quadrature components together.

The selection part of the first phase uses a simple boundary check to choose a symbol from the original constellation S closest to the equalised symbol y_i. However, since the equalised symbols are rotated, they must first be un-rotated before the boundary check is performed. Thereafter, the boundary check is performed on both in-phase and quadrature components of the equalised symbol to determine the location of the closest symbol from S, denoted v, $v \in S$.

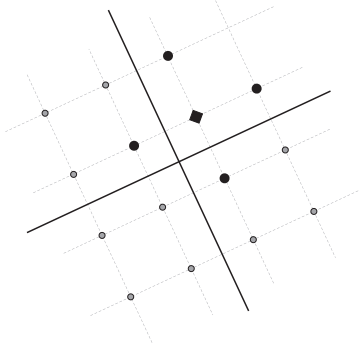

Figure 3: Location of $m = 5$ closest points

The search phase uses the location of symbol v to create a new symbol set $\tilde{T}$ (which is a subset of $\tilde{S}$) consisting of $m-1$ symbols closest to v and v itself. The symbol set $\tilde{T}$ for each symbol v can be pre-determined and stored, and hence need not be recreated for every received symbol. This can be visualized in Figure 3, where $m = 5$ and the diamond represents the location of symbol v after rotation. A conventional ML search is then performed on the new symbol set $\tilde{T}$, as opposed to the entire rotated set $\tilde{S}$. This

greatly reduces the complexity of ML detection required for SSD systems employing large constellations. Note that the original received symbols, and not the equalised symbols, are used for detection.

5. NUMERICAL RESULTS AND SIMULATIONS

Presented in this section are numerical results based on the union bound, NN approximation and the new MDLB, all validated through simulation. The SER performance of the new simplified detection scheme is also compared with that of optimal ML detection. As shown in [5], the optimum rotation angle for both 4-QAM and 16-QAM with a single receive antenna is 31.7°. This rotation angle is used here in all analyses since, as discovered through simulation, the optimum angle for $N \geq 3$ (where N is the number of relays) does not result in a significantly different SER. Simulations are performed over i.i.d flat fading Rayleigh channels with AWGN as described in Section 2 using 4-QAM and 16-QAM modulation. Simulation with 64-QAM modulation is also included for simplified detection with multiple receive antennas. We assume that full CSI for all channels is available at the receiver, that the receive antennas are spaced far enough apart to avoid correlation, and that all systems contain only a single transmit antenna. Since SER vs. rotation angle performance is mirrored horizontally along the 45° line, analysis is restricted to the range 0°-45°.

5.1 *Single Antenna Reception*

We initially analyse SER performance for a system with a single receive antenna. Figure 4 plots simulation results against the conventional union bound, the NN approximation and the new MDLB for both 4-QAM and 16-QAM modulation. The NN approximation is identical to the union bound for 4-QAM and hence is not shown. The new MDLB, while the most accurate at low SNR (below 10dB), is not as tight as the union bound/NN approximation at high SNR at the chosen rotation angle. The NN approximation manages better SER performance than the union bound across the entire SNR range and hence is a better method for approximating the SER. All bounds show better SER performance with a smaller signal set and achieve full diversity.

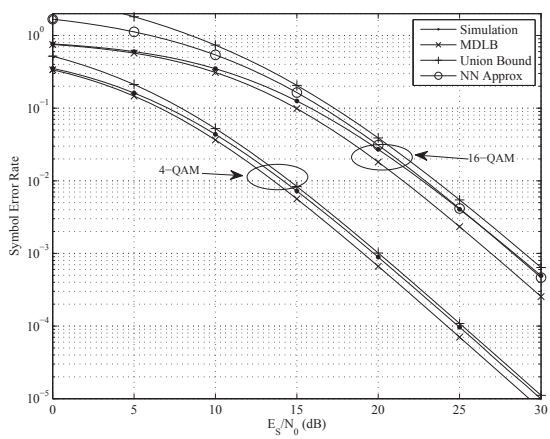

Figure 4: SER of SSD system with $N = 1$

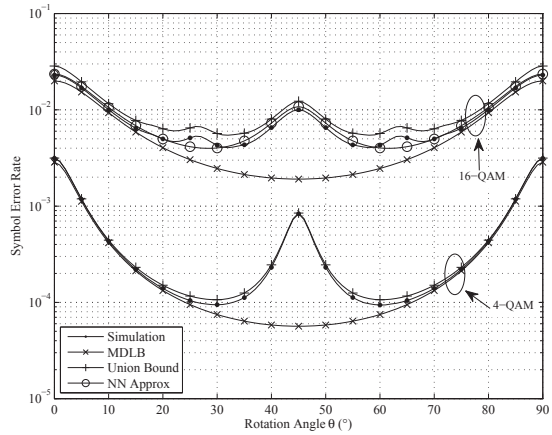

Figure 5: SER vs. rotation angle at 25dB and $N = 1$

As shown in [8], the optimum rotation angle for a system with MRC at the receiver is dependent on the number of receive antennas. It also varies for systems employing coding [14], and for cooperative SSD [18]. It is therefore of interest to determine the error performance of the derived performance bounds relative to the angle of rotation. The SER of the single receive antenna system is plotted against rotation angle in Figure 5 at an SNR of 25dB, well into the high SNR region for both modulation schemes. Inspection of Figure 5 reveals that the new MDLB is tight for the angles between 4° and 24° for 4-QAM and 8° and 20° for 16-QAM, making it suitable for error performance evaluation in systems using rotation angles in the mentioned ranges. Clearly, the NN approximation seems to switch between upper and lower bounding the error probability depending on the rotation angle for 16-QAM; however it does remain fairly tight throughout the rotation angle range. Figure 5 visually corroborates the findings in [5]: the optimum rotation angle, defined as the angle at which the SER is a minimum, is approximately 31.7° for both 4-QAM and 16-QAM when $N = 1$.

Figure 6 plots the SER of the simplified detection scheme for single antenna reception. The simplified detection scheme appears to lose diversity with single antenna reception, yet SER performance is still better than conventional modulation by approximately 7dB at an SER of 10^{-3} for 4-QAM and approximately 4dB at an SER of 10^{-2} for 16-QAM when the number of searches is equal to $m = 3$ and $m = 5$ respectively. Increasing m to 9 brings further performance gains for 16-QAM; the improvement in complexity is worth the slight performance trade-off (approximately 1dB at 10^{-3}) when using 16-QAM and $m = 9$ as compared to conventional SSD detection.

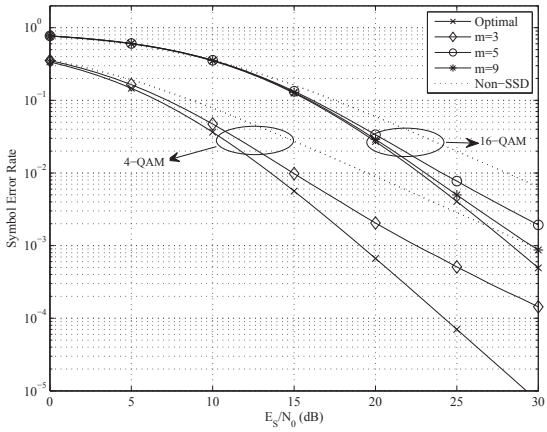

Figure 6: SER of simplified SSD detection scheme with $N = 1$

5.2 Multiple Antenna Reception with MRC

We now consider performance of a system containing multiple receive antennas. The SER of the NN approximation and the new MDLB are plotted against simulation results in Figure 7 for $N = 3$ and $N = 4$ receive antennas and MRC reception. The new MDLB, while not as tight as the NN approximation in the high SNR region, is far more accurate in the low SNR region and tight enough across the SNR range for the evaluation of SER performance in systems containing multiple receive antennas.

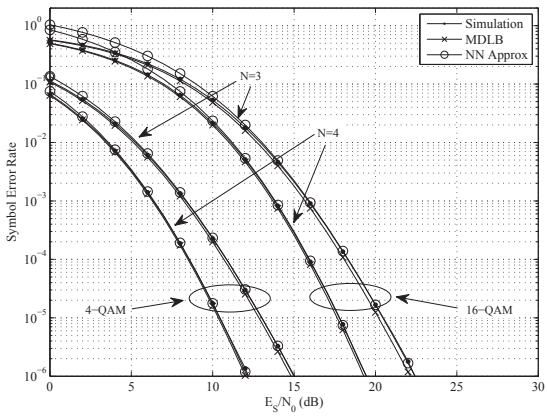

Figure 7: SER of multiple receive antenna SSD system with MRC reception

Figure 8 plots SER against rotation angle for the NN approximation and the MDLB at 10dB for 4-QAM and 15dB for 16-QAM with $N = 3$ receive antennas. The NN approximation is clearly tighter than the MDLB since the system is well into the high SNR region for both modulations. The MDLB, however, manages to be tight for a wide range of rotation angles: between 4° and 28° for 4-QAM and between 8° and 24° for 16-QAM. Even though the MDLB appears to only be tight for the mentioned rotation angles as shown in Figure 8, it is suitable for SER evaluation in multiple receive antenna systems as shown in Figure 7. Comparing Figure 4 and Figure 7, it is clear that the MDLB achieves tighter performance across the SNR range when $N \geq 3$ compared to $N = 1$, and comparing Figure 5 and Figure 8 shows that the MDLB achieves tighter performance across a wider rotation angle range when $N = 3$ compared to $N = 1$.

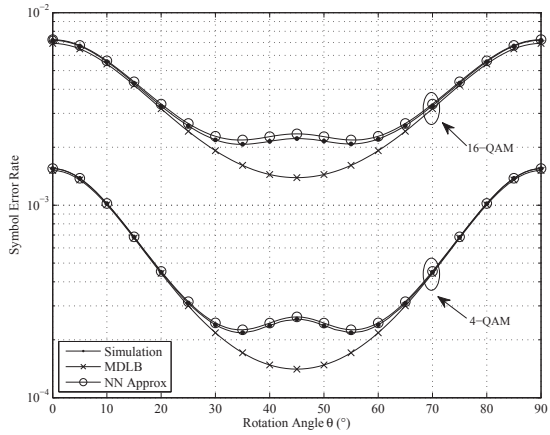

Figure 8: SER vs. rotation angle with $N = 3$

Figure 9 plots the SER of the simplified detection scheme for $N = 3$ and $N = 4$ receive antennas. SER performance in all cases is close to indistinguishable from the optimal detection scheme. Thus, in a system with multiple receive antennas, the simplified detection scheme is able to achieve very close to optimal detection SER performance with greatly reduced complexity compared to the optimal detection scheme. This is achieved for both $N = 3$ and $N = 4$ with 4-QAM when $m = 3$; however with 16-QAM and 64-QAM, $m = 5$ can only achieve close to optimal SER performance when $N = 4$, $m = 9$ is required when $N = 3$. Increasing the number of receive antennas results in a larger received SNR; enabling the value of m to be reduced since the received symbols have a higher probability of being closer in Euclidean distance to the transmitted symbols.

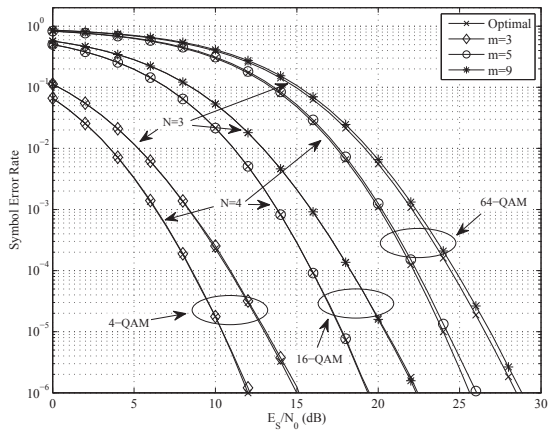

Figure 9: SER of simplified SSD detection scheme with multiple receive antennas and MRC

Table 1: Average number of searches for given value of m, corresponding average complexity C and corresponding complexity reduction percentage

	4-QAM			16-QAM			64-QAM		
	Avg.	C	%	Avg.	C	%	Avg.	C	%
Optimal ML	4	52	100%	16	208	100%	64	832	100%
$m=3$	3	49	5.77%	-	-	-	-	-	-
$m=5$	-	-	-	4	62	70.19%	4.5	68.5	91.77%
$m=9$	-	-	-	6.25	91.25	56.13%	7.56	108.28	86.99%

5.3 *Simplified Detection Scheme Complexity Analysis*

Finally, we compare the complexity reduction achieved in terms of the number of real addition and multiplication operations needed by the simplified detection scheme as compared to optimal ML decoding in Table 1. This is determined by computing the number of operations needed per search, as well as the number of operations needed for the equalisation and rotation operations which are part of the first equalisation and selection phase. For the optimal ML detection scheme, the number of searches is the same for any point in the constellation. However, for the simplified detection scheme the number of searches required is dependent on the location of symbol v, we thus compute the average number of searches required for a given M-QAM constellation size and value of m.

The average number of searches required for square M-QAM constellations and $M > 4$ can be easily shown, by considering the number of searches required for corner, side, and centre constellation points, to be:

$$A = \left\{ \frac{1}{M} \begin{pmatrix} 4\iota + \left(4\sqrt{M} - 8\right)\upsilon + \\ \left(M - 4\sqrt{M} + 4\right)m \end{pmatrix} : m = 5, 9 \right\} \quad (24)$$

where A is the average number of searches required, $\iota = \lfloor \sqrt{m} + 1 \rfloor$, $\upsilon = \lceil \frac{2}{3} m \rceil$, and $\lfloor \cdot \rfloor$ and $\lceil \cdot \rceil$ refer to the floor and ceiling functions respectively.

We have modelled the fading to be Rayleigh distributed, hence the equalisation operation is equivalent to the division of a complex number by a real number. For the purposes of comparison, we will consider a real division operation to be equivalent to two real multiplication operations; hence the equalisation operation consists of four real multiplication operations. The rotation operation consists of four real multiplications and two real additions, giving a total of 6 operations. The complexity of a single search can be found by examining equation (6). Without any difficulty, it can be seen that each symbol requires ten real multiplications and three additions, giving a total of thirteen operations per symbol per search.

Considering the above, we can write a simple complexity equation for the simplified detection scheme:

$$C = 4 + 6 + 13A \quad (25)$$

where C gives the complexity of the simplified detection scheme in terms of the number of real addition and multiplication operations required. The percentage reduction achieved is given by $\% = \frac{opt - simp}{opt} \times 100$, where opt refers to the number of operations needed for optimal ML detection and $simp$ refers to the number of operations needed for the simplified detection scheme.

For 4-QAM, the achieved SER for simplified detection with single antenna reception is in between non-SSD transmission and optimal detection while giving a complexity reduction of 5.77%. However, with multiple receive antennas, SER performance is close to indistinguishable from optimal detection with only 3 searches required. For 16-QAM, m can be reduced to 5 when $N = 4$ and reduced to 9 when $N = 3$, resulting in complexity reductions of 70.19% and 56.13% respectively. Close to optimal SER performance can be achieved using simplified detection with $m = 9$, 16-QAM, and a single receive antenna (up to 1dB at an error rate of 10^{-3}). This results in a complexity reduction of 56.13%, however diversity from SSD is lost. Finally, similar to 16-QAM, m can be reduced to 5 when $N = 4$ and reduced to 9 when $N = 3$, resulting in complexity reductions of 91.77% and 86.99% respectively for 64-QAM with multiple receive antennas. With all modulation schemes, increasing the number of receive antennas improves the SER performance of the simplified detection scheme and brings SER performance closer to that of optimal detection.

6. CONCLUSION

SER performance of an SSD system employing a single transmit antenna and N receive antennas with MRC reception was presented. The union bound and the NN approximation were presented in closed form along with a new lower bound based on the minimum Euclidean distance (MDLB) of a rotated constellation. This was also presented in closed form. It was found that the NN approximation exhibits tighter SER performance than the union bound across most of the rotation angle range, and that the new MDLB exhibits tighter low SNR performance than the upper bounds, yet is looser than the upper bounds at high SNR and angles closer to the optimum when $N = 1$. However, when $N \geq 3$, SER performance of the MDLB is tight across the SNR range. The MDLB is simple to compute and only requires a change in variable to evaluate SSD in other systems if the conventional M-QAM conditional SER is used.

A simplified detection scheme for SSD systems was also presented and compared to the optimal detection scheme using simulation. It was found that the simplified detection scheme loses diversity when $N = 1$, however SER performance is still significantly better than non-SSD transmission and improves with increasing values of m. With $N = 3$, SER performance is close to indistinguishable from optimal detection for 4-QAM, 16-QAM and 64-QAM with complexity reductions of 5.77%, 56.13% and 86.99% respectively; and with $N = 4$, of 5.77%, 70.19% and 91.77% respectively.

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